Thermal property reconstruction via reducing ultrasonic temperature measurement noise using polynomial fitting method

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Targets: Thermal properties such as thermal conductivity, thermal capacity, thermal diffusivity; and thermal quantities such as thermal source/sink and perfusion.

From only temperature distribution data, the targets are reconstructed in vivo.

- US measurement
- MRI measurement
- Infrared imaging
- Pyroelectric sensing

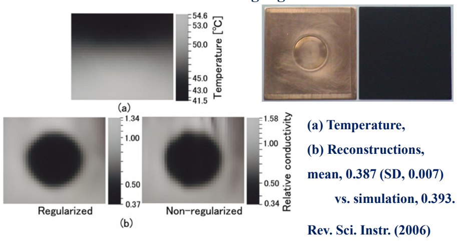
Applications:

1. Monitoring, planning and controlling various thermal coagulation treatments of human diseases such as HIFU, rf electromagnetic wave, microwave etc, various hyperthermia modalities, cryotherapy, chemotherapy, anti-cancer drug
2. Non-invasive measurements of living things, properties/structures, function, thermal conductivity path etc.
3. Non-invasive examination of thermal object, substance, structure etc.
4. Monitoring of these generations and growths etc.

Infra-red thermography

Copper plate (50 mm × 50 mm), ROI size: 37.7 mm × 32.0 mm

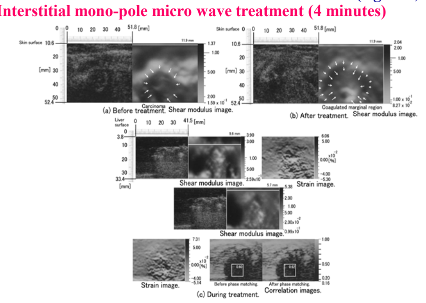
Thickness: 0.25 mm in the central circular region (Rad.: 10.0 mm) and 1.0 mm in the surrounding region



(a) Temperature,
(b) Reconstructions,
mean, 0.387 (SD, 0.007)
vs. simulation, 0.393.

Rev. Sci. Instr. (2006)

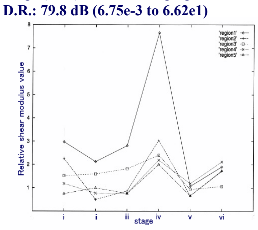
Shear modulus reconstruction on human *in vivo* liver (Age: 73, Female) – Interstitial mono-pole micro wave treatment (4 minutes)



C. Sumi, IEEE Trans UFFC (2005)

Dependency of shear modulus on temperature can also be used for temperature measurement.

Stage vs. shear modulus value
Local region size: 8.3 mm x 10.0 mm
Dynamic range (D.R.): 69.8 dB ranging from 3.71e-3 to 11.4
cf. Image D.R.: 79.8 dB (6.75e-3 to 6.62e1)



C. Sumi, IEEE Trans UFFC (2005)

Stage i:
before being heated
D.R.: 29.7 dB
(1.91e-1 to 5.82)

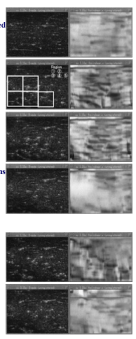
Stage ii:
heated for 5 mins
53.5 dB
(1.91e-2 to 9.02)

Stage iii:
left for 1min
69.7 dB
(3.71e-3 to 11.4)

Stage iv:
left for more 9 mins
35.1 dB
(1.27e-1 to 7.17)

Stage v:
heated for 5 mins
46.2 dB
(2.70e-2 to 5.51)

Stage vi:
left for 10 min
54.5 dB
(9.19e-3 to 4.89)



Temperature measurement (Ebbini's model)

E. S. Ebbini etal., IEEE Trans. Ultrasonic, Ferroelectrics, and Frequency Control, vol. 45, no.4, pp. 1088-1099, July 1998.

Change in temperature at depth **z**

$$\Delta T(z) = \frac{c(T_0)}{2} k \frac{\partial}{\partial z}(\delta t(z)),$$

where $\delta t(z)$ is temporal shift due to the temperature change,

c: Sound speed

thermal parameter: $k = 1/(\beta - \alpha)$

$\alpha(z)$: Coefficient of change in volume due to temperature change

$\beta(z)$: Coefficient of c due to temperature change.

Measurement of thermal parameter: C. Sumi et al, JJAP (2007)

Reconstruction

Together with thermal conductivity k, thermal capacity ϱc, thermal diffusivity ϰ, perfusion and thermal source are reconstructed.

- Simultaneous 1st order partial differential equations:

$$\nabla T_i \bullet \nabla k + k\nabla^2 T_i + Q_1 - Q_2 = \rho c \frac{d}{dt} T_i \quad \text{in } \Phi \text{ (ROI)}$$

Q_1 : thermal source Q_2 :perfusion

Pennes' model $wc_b(T - T_b)$

w: perfusion coefficient, cb: specific heat, Tb: blood temperature

- Initial condition:

$$k(x,y,z) = k_l(x,y,z),\ (x,y,z) \in \phi_{kl}\ (l = 1 \sim Nk)$$

and/or

$$\rho c(x,y,z) = \rho_l c_l(x,y,z),\ (x,y,z) \in \phi_{\rho cl}\ (l = 1 \sim N\rho c)$$

$\phi_{kl}, \phi_{\rho cl} (\subset \Phi)$: reference region.

C. Sumi, Phys. Med. Biol. (2007)

Regulation

Using regulation parameter a, the order of eigenvalues of partial matrices are set to almost same as follows,

$$\begin{pmatrix} A/a & B & C \end{pmatrix} \begin{pmatrix} aw \\ k \\ \rho c \end{pmatrix} = \vec{D}.$$

The above-equations are solved for aw, k and ρc.

Perfusion w is determined by dividing aw by a.

Bio-heat transfer equation with no perfusion and no thermal source

Thermal equation:

$$\rho c \frac{dT}{dt} = \nabla \cdot (k \nabla T) + Q$$

where T : temperature

k : thermal conductivity

ρ : density

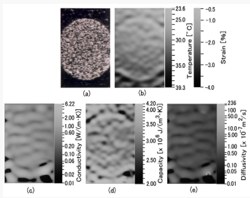
c : specific heat

C: thermal capacity (=ρc)

Q: thermal source or thermal sink

LPF

Target: An agar phantom (height, 50 mm) with a circular region (dia. = 15 mm; depth 30 mm) having different thermal properties realized by adding a small amount of copper powder: A ROI was set (19.8 mm x 13.2 mm spreading out from a depth of 19.7 to 39.5 mm). (a) B-mode image, and images of (b) measured temperature, strain (thermal parameter, –458 ℃), and reconstructed (c) conductivity k, (d) capacity ρc and (e) diffusivity k/ρc.



- Table. Means of reconstructed k, ρc and k/(ρc) evaluated at central square region (6.5 × 5.8 mm) at circular region.

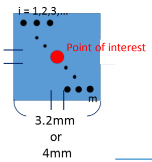
Value	$k\ [W/(m\cdot K)]$	$\rho c\ (\times 10^6)\ [J/(m^3\cdot K)]$	$k/\rho c\ (\times 10^{-7})\ [m^2/s]$
Ref.	0.625	4.20	1.49
Mean	0.700	3.18	2.20

C. Sumi, Phys. Med. Biol. (2007)

Polynomial fitting

Temperature distribution is approximated to be expressed by parabolic functions.

$$T = ax^2 + by^2 + cxy + dx + ey + f$$



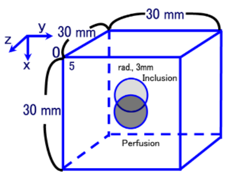
A square or a circle region is set at each position in an ROI, and the temperature of center is estimated by fitting temperatures Ti (I = 1 to m) to the parabolic function.

A side of square or a diameter, 3.2 mm or 4 mm,
Compared with a width of LPF, 3.2mm.

Polynomial fitting for strain measurement: Kallel F., Ophir J. 1997. A least-squares strain estimator for elastography. Ultrason. Imaging 19, 195–208; for temperature measurement: Xu Y et al. 2009. Temperature-change-based thermal tomography. Int. J. of Biomed Imag 2009, Article ID 464235.

Simulated reconstructions including perfusion

—Upper surface of phantom is warmed from 36 to 42℃.

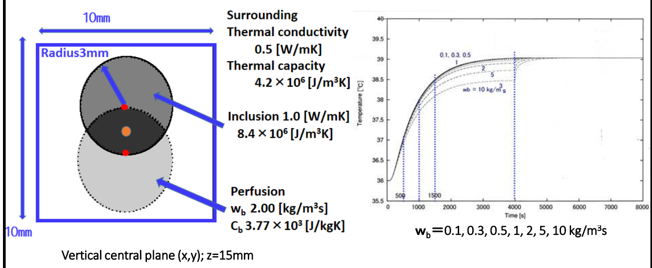


Region of interest ROI:
Cube (10 mm sides) centered on phantom

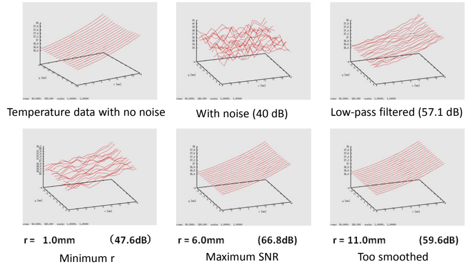
Initial temperature:
36℃ (whole body)

Perfusion (blood flow):
Temperature Tb = 36℃
Specific heat cb = 3,770 J/Kkg

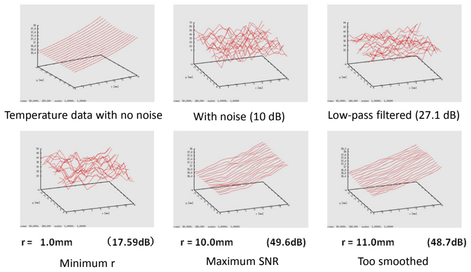
Temporal change in temperature (perfusion stopped at 4,000 sec)



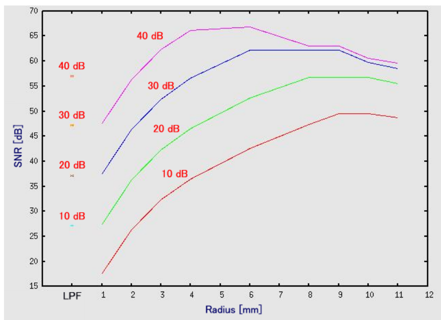
SN ratio 40 dB



SN ratio 10 dB



Radius r vs SNR



Reconstructions

Reconstruction equations（finite difference approximation）

$$(A\ \ B)\begin{pmatrix}k\\C\end{pmatrix}=d$$

k: thermal conductivity distribution, C: thermal capacity distribution

Regularizations on plural thermal properties

Least squares solution, i.e., minimization of error energy:

$$E\left(\begin{pmatrix}k\\C\end{pmatrix}\right)=\left(d-(A\ \ B)\begin{pmatrix}k\\C\end{pmatrix}\right)^T\left(d-(A\ \ B)\begin{pmatrix}k\\C\end{pmatrix}\right)+\alpha_k|D^TDk|^2+\alpha_C|D^TDC|^2\xrightarrow{(k,C)}\text{Min.}$$

$$\begin{pmatrix}k\\C\end{pmatrix}=\left(\begin{pmatrix}A^T\\B^T\end{pmatrix}(A\ \ B)+\begin{pmatrix}\alpha_kD^TD & 0\\0 & \alpha_CD^TD\end{pmatrix}\right)^{-1}(A\ \ B)^Tb,$$

where **D**: Gradient operator

α_k: Regularization parameter for thermal conductivity reconstruction

α_C: Regularization parameter for thermal capacity reconstruction

Agar phantom experiment

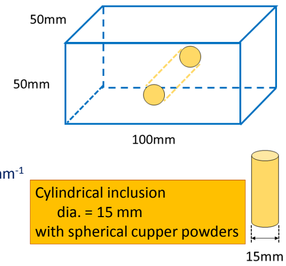
Agar phantom
50×50×100mm³
Concentration, 3.87%

Whole body with initial temperature, 21.4℃
warmed from lower surface
using hot water 46℃.

ROI: Rectangular (19.8×13.2mm²)
centered on cylindrical inclusion

δt(z) measured using 2D cross-spectrum
phase gradient method (CSPGM)
Cutoff frequency of differential filter, 0.511mm⁻¹

US equipment, AlokaSSD5500
US frequency, 7.5MHz
frame rate, 0.05Hz。



Cylindrical inclusion
dia. = 15 mm
with spherical cupper powders

LPF

Target: An agar phantom (height, 50 mm) with a circular region (dia. = 15 mm; depth 30 mm) having different thermal properties realized by adding a small amount of copper powder: A ROI was set (19.8 mm x 13.2 mm spreading out from a depth of 19.7 to 39.5 mm). (a) B-mode image, and images of (b) measured temperature, strain (thermal parameter, −458 ℃), and reconstructed (c) conductivity k, (d) capacity ρc and (e) diffusivity k/ρc.

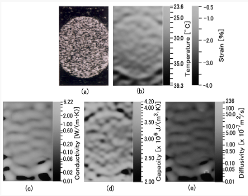


Table. Means of reconstructed k, ρc and k/(ρc) evaluated at central square region (6.5 × 5.8 mm) at circular region.

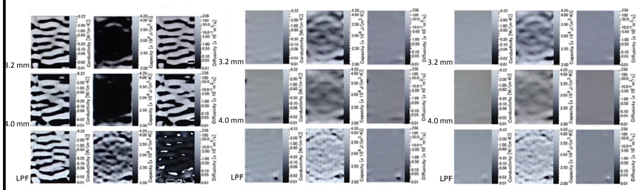
Value	k $[W/(m \cdot K)]$	ρc $(\times 10^6)$ $[J/(m^3 \cdot K)]$	$k/\rho c$ $(\times 10^{-7})$ $[m^2/s]$
Ref.	0.625	4.20	1.49
Mean	0.700	3.18	2.20

C. Sumi, Phys. Med. Biol. (2007)

Fixed regularization parameter $\alpha_C = 1 \times 10^{-13}$

Changed α_k:

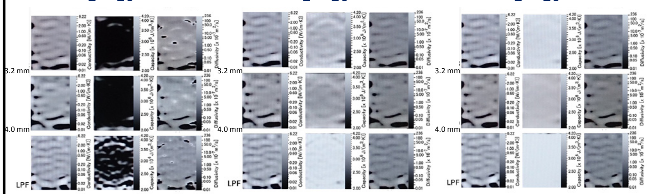
1×10^{-7} 5×10^{-6} 1×10^{-5}



Fixed $\alpha_k = 1 \times 10^{-6}$

Changed α_C

1×10^{-14} 1×10^{-12} 1×10^{-11}



The best results:

the same parameter values as those of LPF case:

$\alpha_k = 1 \times 10^{-6}$

$\alpha_C = 1 \times 10^{-13}$

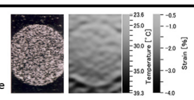
Ref.

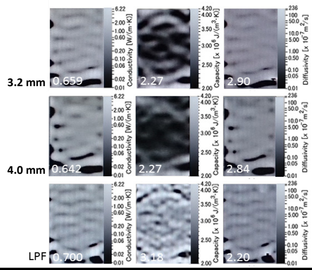
Thermal conductivity : 0.625[w/(m・k)]

Capacity : 4.20×10^6 [J/(m^3・k)]

Diffusivity : 1.49×10^{-7} [m^2 /s]

B-mode

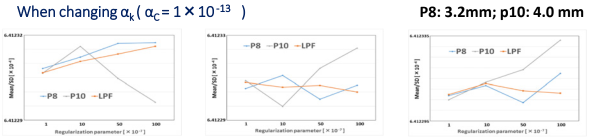




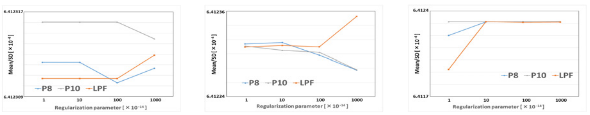
SN ratio（＝mean/SD）

When changing α_k ($\alpha_C = 1 \times 10^{-13}$)

P8: 3.2mm; p10: 4.0 mm



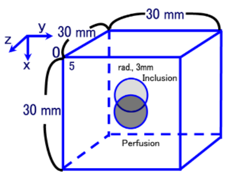
When changing α_C ($\alpha_k = 1 \times 10^{-6}$)



Thermal conductivity Thermal capacity Thermal diffusivity

Simulated reconstructions including perfusion

—Upper surface of phantom is warmed from 36 to 42℃.



Region of interest ROI :

Cube (10 mm sides) centered on phantom

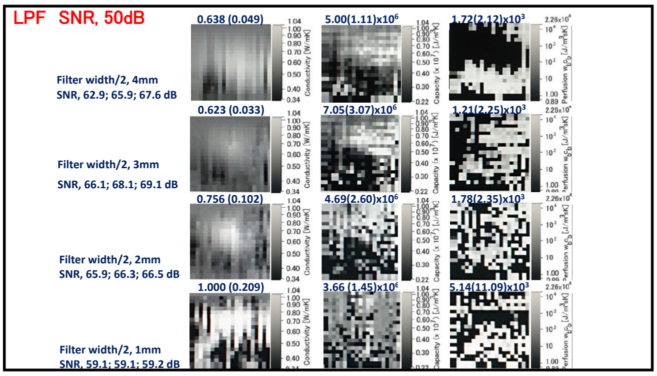
Initial temperature :

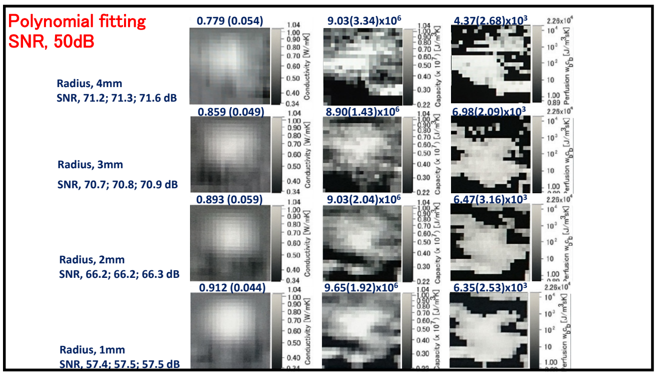
36℃ (whole body)

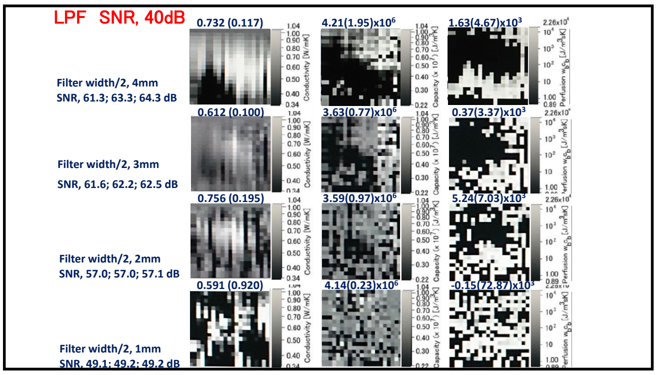
Perfusion (blood flow) :

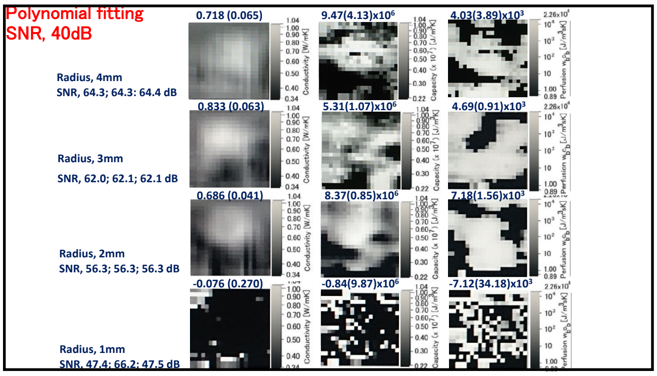
Temperature Tb = 36℃

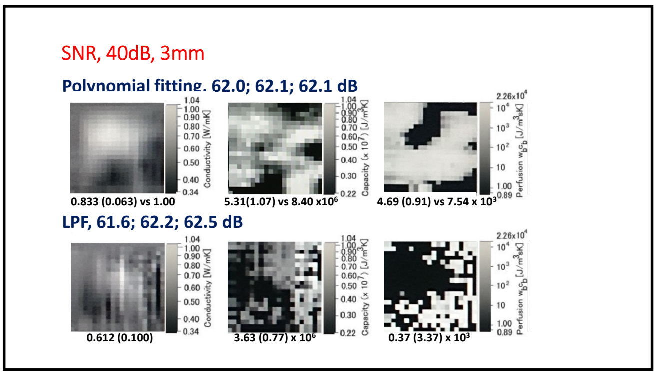
Specific heat cb = 3,770 J/Kkg

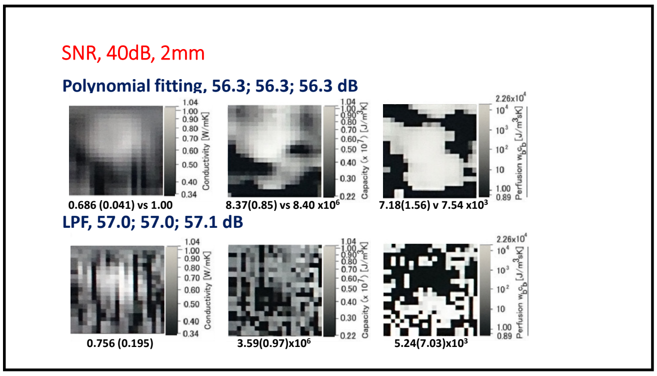

LPF SNR, 50dB
Filter width/2, 4mm
SNR, 62.9; 65.9; 67.6 dB
0.638 (0.049)
5.00(1.11)x10^6
1.72(2.12)x10^3
Filter width/2, 3mm
SNR, 66.1; 68.1; 69.1 dB
0.623 (0.033)
7.05(3.07)x10^6
1.21(2.25)x10^3
Filter width/2, 2mm
SNR, 65.9; 66.3; 66.5 dB
0.756 (0.102)
4.69(2.60)x10^6
1.78(2.35)x10^3
Filter width/2, 1mm
SNR, 59.1; 59.1; 59.2 dB
1.000 (0.209)
3.66 (1.45)x10^6
5.14(11.09)x10^3


Polynomial fitting
SNR, 50dB
Radius, 4mm
SNR, 71.2; 71.3; 71.6 dB
0.779 (0.054)
9.03(3.34)x10^6
4.37(2.68)x10^3
Radius, 3mm
SNR, 70.7; 70.8; 70.9 dB
0.859 (0.049)
8.90(1.43)x10^6
6.98(2.09)x10^3
Radius, 2mm
SNR, 66.2; 66.2; 66.3 dB
0.893 (0.059)
9.03(2.04)x10^6
6.47(3.16)x10^3
Radius, 1mm
SNR, 57.4; 57.5; 57.5 dB
0.912 (0.044)
9.65(1.92)x10^6
6.35(2.53)x10^3


LPF SNR, 40dB
Filter width/2, 4mm
SNR, 61.3; 63.3; 64.3 dB
0.732 (0.117)
4.21(1.95)x10^6
1.63(4.67)x10^3
Filter width/2, 3mm
SNR, 61.6; 62.2; 62.5 dB
0.612 (0.100)
3.63(0.77)x10^6
0.37(3.37)x10^3
Filter width/2, 2mm
SNR, 57.0; 57.0; 57.1 dB
0.756 (0.195)
3.59(0.97)x10^6
5.24(7.03)x10^3
Filter width/2, 1mm
SNR, 49.1; 49.2; 49.2 dB
0.591 (0.920)
4.14(0.23)x10^6
-0.15(72.87)x10^3


Polynomial fitting
SNR, 40dB
Radius, 4mm
SNR, 64.3; 64.3: 64.4 dB
0.718 (0.065)
9.47(4.13)x10^6
4.03(3.89)x10^3
Radius, 3mm
SNR, 62.0; 62.1; 62.1 dB
0.833 (0.063)
5.31(1.07)x10^6
4.69(0.91)x10^3
Radius, 2mm
SNR, 56.3; 56.3; 56.3 dB
0.686 (0.041)
8.37(0.85)x10^6
7.18(1.56)x10^3
Radius, 1mm
SNR, 47.4; 66.2; 47.5 dB
-0.076 (0.270)
-0.84(9.87)x10^6
-7.12(34.18)x10^3


SNR, 40dB, 3mm
Polynomial fitting, 62.0; 62.1; 62.1 dB
0.833 (0.063) vs 1.00
5.31(1.07) vs 8.40 x10^6
4.69 (0.91) vs 7.54 x 10^3
LPF, 61.6; 62.2; 62.5 dB
0.612 (0.100)
3.63 (0.77) x 10^6
0.37 (3.37) x 10^3
Conductivity [W/mK]
Capacity (x 10^7) [J/m^3K]
Perfusion $w_b c_b$ [J/m^3sK]


SNR, 40dB, 2mm
Polynomial fitting, 56.3; 56.3; 56.3 dB
0.686 (0.041) vs 1.00
8.37(0.85) vs 8.40 x10^6
7.18(1.56) v 7.54 x10^3
LPF, 57.0; 57.0; 57.1 dB
0.756 (0.195)
3.59(0.97)x10^6
5.24(7.03)x10^3
Conductivity [W/mK]
Capacity (x 10^7) [J/m^3K]
Perfusion $w_b c_b$ [J/m^3sK]

Conclusions

Reduction effects of temperature noise data by using polynomial fitting method on thermal reconstructions were confirmed. In agar phantom experiments, thermal conductivity and capacity were reconstructed, whereas in simulations, reconstructions including perfusion were performed.

- With a reasonably large radius r, a higher SNR of temperature data were obtained than with LPF. Using a too large radius r, SNR became low due to too much smoothing.

e.g., For simulated raw temperature data with SNR of 40 dB, the highest SNR of 64 dB was achieved with a radius 4mm. The result of LPF was 61 dB.

- Accordingly, a more accurate and more stable reconstruction was obtained than with LPF. With respect to the thermal property reconstructions, the smaller radius than the above-mentioned radius 4mm suffered from the effects of too much smoothing.

e.g., For raw data with 50 and 40 dB, the best radii were 1mm and 2mm, respectively.

The more accurate reconstructions were obtained for raw temperature data with the higher SNR; and the best radius was smaller.

- Experimental results on perfusion reconstruction and using temperature estimated from shear modulus reconstruction will be reported elsewhere in the near future. Temperature measurement on the basis of shear modulus reconstruction (see USA patents etc) is more robust to tissue motion artifact than that on the basis of dependency of US propagation on temperature change.